Technical Comments

Comment on "Average Relative Velocity of Sporadic Meteoroids in Interplanetary Space"

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In a recent Technical Note, Kessler[1] derived an expression for the mass-cumulative influx of meteoroids which was proportional to $m^{0.9(-1.34)}$ from the expression $\propto I^{-1.34}$ for the intensity-cumulative distribution of meteors, representing Hawkins and Upton's[2] results from the statistical analysis of meteor magnitude. The 10% disparity in the exponents of meteoroid mass m and meteor standardized maximum luminous I resulted from Kessler's[1] use of the result of a least squares analysis of meteor data by Jacchia, Verniani, and Briggs,[3] showing $I \propto m^{0.9} V_\infty^{3.5}$. But the values for mass m for those results presupposed that meteor luminous efficiency was $\propto V_\infty$ and functionally independent of meteoroid mass m. The indicated disparity in the exponents of I and m in the corresponding cumulative distributions actually vanishes when mass m is recomputed with the luminous efficiency from the 1958 physical theory of meteors by Öpik.[4]

Because the product of luminous efficiency and kinetic energy is $\propto \int I dt$ invariantly between alternative theory for "photometric" mass, the above results support the substitution $m \propto m_B \beta_B V_\infty^{-1}$ where m_B and β_B are, respectively, mass and luminous efficiency by Öpik's[4] theory. Therefore, if β_B is such a function of m_B that, at constant velocity, it has roughly the effect of an exponential role $\beta_B \propto m_B^X$, then the exponent of m_B in the mass-cumulative distribution is $0.9(1 + x)(-1.34)$ instead of $0.9(-1.34)$. The effective value of x can be approximated by the partial derivative of log β_B with respect to log m_B. Öpik[5] gave a table of values to illustrate his 1958 theory given in Ref. 4 and Dalton[6] approximated them by a mathematical model which is easier to program than the theory directly. For dustball meteoroids in the mass interval $0.1 \leqq m_B \leqq 110\, g$,

$$\log \beta_B = -10^{a_0}(-\log m_B\beta_B)^{a_1} \tag{1}$$

where, in km/sec and base ten,

$$a_0 = 0.4042 + 0.193\ (\log \log V_\infty - 0.0682) \quad \text{for } V_\infty \geqq 14.8$$

$$a_0 = 0.4042 + 0.33\ (0.0682 - \log \log V_\infty) \quad \text{for } V_\infty < 14.8$$

and

$$a_1 = 0.0840 + 0.00113|V_\infty - 14.8| \tag{2}$$

Then,

$$\begin{aligned} x &= \partial \log \beta_B/\partial \log m_B \\ &= -(a_1 \log \beta_B)/[(a_1 - 1) \log \beta_B - \log m_B] \\ &= a_1/(1 - a_1) \text{ when } m_B = 1 \text{ g} \end{aligned} \tag{3}$$

Equations (2) and (3) give x as only a weak function of V_∞, so that, for a 1 g particle, the value of x in Eq. (3) just increases from 0.0974 at 19 km/sec to 0.1 at 29 km/sec. This example illustrates that the exponent $0.9(1 + x)(-1.34)$ of m_B in the cumulative distribution of m_B is essentially the same as the exponent -1.34 of I in the cumulative distribution of I.

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References

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[2] Hawkins, G. S. and Upton, E. K. L., "The Influx Rate of Meteors in the Earth's Atmosphere," *The Astrophysical Journal*, Vol. 128, No. 3, 1958, pp. 727–735.

[3] Jacchia, L. G., Verniani, F., and Briggs, R. E., "An Analysis of the Atmospheric Trajectories of 413 Precisely Reduced Photographic Meteors," *Smithsonian Contributions to Astrophysics*, Vol. 10, No. 1, Smithsonian Institution, Washington, D. C., 1967, pp. 1–139.

[4] Öpik, E. J., *Physics of Meteor Flight in Atmosphere*, Interscience, New York, 1958.

[5] Öpik, E. J., "Tables of Meteor Luminosities," *The Irish Astronomical Journal*, Vol. 6, No. 1, March 1963, pp. 3–11.

[6] Dalton, C. C., "The Masses of Meteors and the Selection of a Representative Data Sample," TM X-53760, 1967, NASA, pp. 63–73.

Comment on "A Second-Order Correction to the Glauert Wall Jet Solution"

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Recently Plotkin[1] derived the following equations for the second-order effect of the straight wall jet:

$$g''' + fg'' + 5f'g' = 0,\ g(0) = g'(0) = 0,\ g'(\infty) = -1 \tag{1}$$

Here f is the solution of the equations for the first-order effect, as follows:

$$f''' + ff'' + 2(f')^2 = 0,\ f(0) = f'(0) = 0,\ f'(\infty) = 0 \tag{2}$$

Glauert[2] showed that $f(\infty) = 1$ and

$$\eta = \ln[(1 + x + x^2)^{1/2}/(1 - x)] + \alpha \arctan[\alpha x/(2 + x)] \tag{3}$$

with $x = f^{1/2}$ and $\alpha = 3^{1/2}$. Plotkin solved Eqs. (1) by numerical calculations. Here analytical form of the solution is derived. Putting $\varphi = g'$, and using the relation $f' = \frac{2}{3}(f^{1/2} - f^2)$, Eqs. (1) are reduced to

$$(d^2\varphi/dx^2) + [30x/(1 - x^3)]\varphi = 0,\ \varphi(0) = 0,\ \varphi(1) = -1 \tag{4}$$

Noting that the transformation $\xi = x^3$ reduces the first of Eqs. (4) to a hypergeometric differential equation,

$$\xi(1 - \xi)(d^2\varphi/d\xi^2) + \tfrac{2}{3}(1 - \xi)(d\varphi/d\xi) + \tfrac{10}{3}\varphi = 0$$

it is easy to show that a solution of Eq. (4) is given by

$$\varphi_1 = (1 - x^3)(1 - 4x^3) \tag{5}$$

Another linearly independent solution, φ_2, is obtained by the d'Alembert method of depression as

$$\varphi_2 = 3x(17 - 20x^3) + 10\varphi_1\{\ln[(1 + x + x^2)^{1/2}/(1 - x)] + \alpha \arctan[(1 + 2x)/\alpha]\} \tag{6}$$

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